WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2017
TEST 2: Functions

Name: _____

Monday 3rd April

Time: 50 minutes **Mark** **/40 =** **%**

- Answer all questions neatly in the spaces provided. **Show all working.**
- You are permitted to use the Formula Sheet in **both** sections of the test.
- You are permitted one A4 page (one side) of notes in the calculator assumed section.

Ave = 32.6
≈ 81.5%.

Calculator free section **Suggested time: 30 minutes** **/26**

9

1. [10 marks]

Two functions f and g are defined by $f(x)=\sqrt{x+4}$ and $g(x)=e^x-1$

a) Express $g \circ f(x)$ in terms of x

$= e^{\sqrt{x+4}} - 1$

[1]

b) What is the natural domain of $y = g \circ f(x)$

$\mathbb{R}, \; x+4 \geq 0 \Rightarrow x \geq -4$

[2]

c) What is the range (co-domain) of $y = g \circ f(x)$

$\mathbb{R}, \; y \geq 0$

[2]

A third function $y = h(x)$ is such that $f(h(x)) = \sqrt{x^2-4}$.

d) Express $h(x)$ in terms of x.

$h(x) = x^2 - 8$

[1]

e) Clearly define $y = f^{-1}(x)$ and specify both its domain and range.

$x = \sqrt{y+4}$

$\Rightarrow y = x^2 - 4$ D: $\mathbb{R}, \; x \geq 0$

R: $\mathbb{R}, \; y \geq -4$

[4]

3

2. [7 marks]

a) Solve the inequality $|2-x| \geq 5$

$x \geq 5$ units from 2

$\therefore x \leq -3$ or $x \geq 7$

if $x \geq 2$ $\quad -(2-x) \geq 5$

$x \geq 7$

if $x < 2$ $\quad 2-x \geq 5$

$\therefore x \leq -3$

or solve $|2-x| = 5$ by squaring $(2-x)^2 = 25$

[3]

b) Calculate where the line $y = |2x-6|$ intersects $y = |x+2|+1$ and illustrate your solution on the axes provided.

$2|x-3| = |x+2| + 1$

$x \geq 3$ $\quad 2(x-3) = x+2+1$

$\therefore x = 9$

$-2 \leq x < 3$ $\quad -2(x-3) = x+2+1$

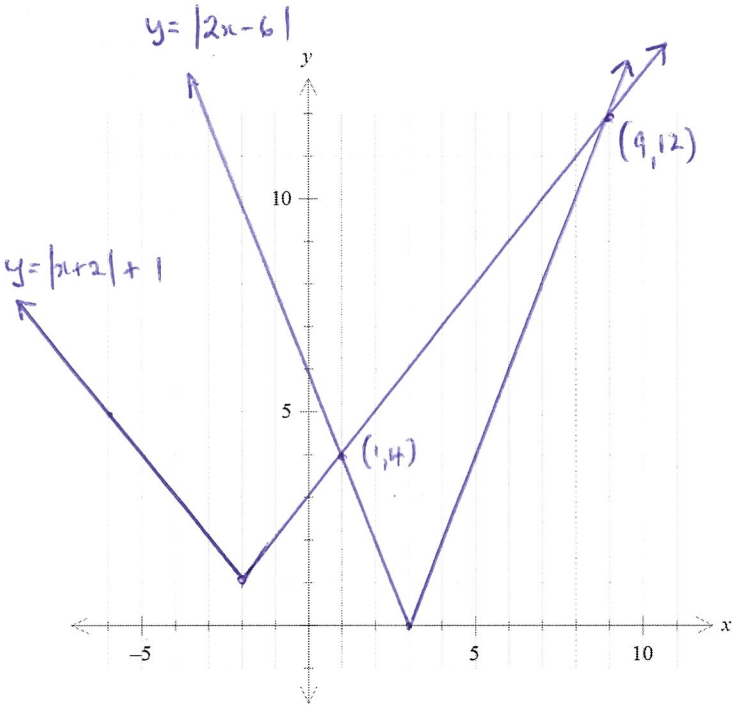
$\therefore 3x = 3$

$x = 1$

$x < -2$ $\quad -2(x-3) = -x-2+1$

$x = 7$ X

$\therefore$ intersect at $(9,12)$ & $(1,4)$



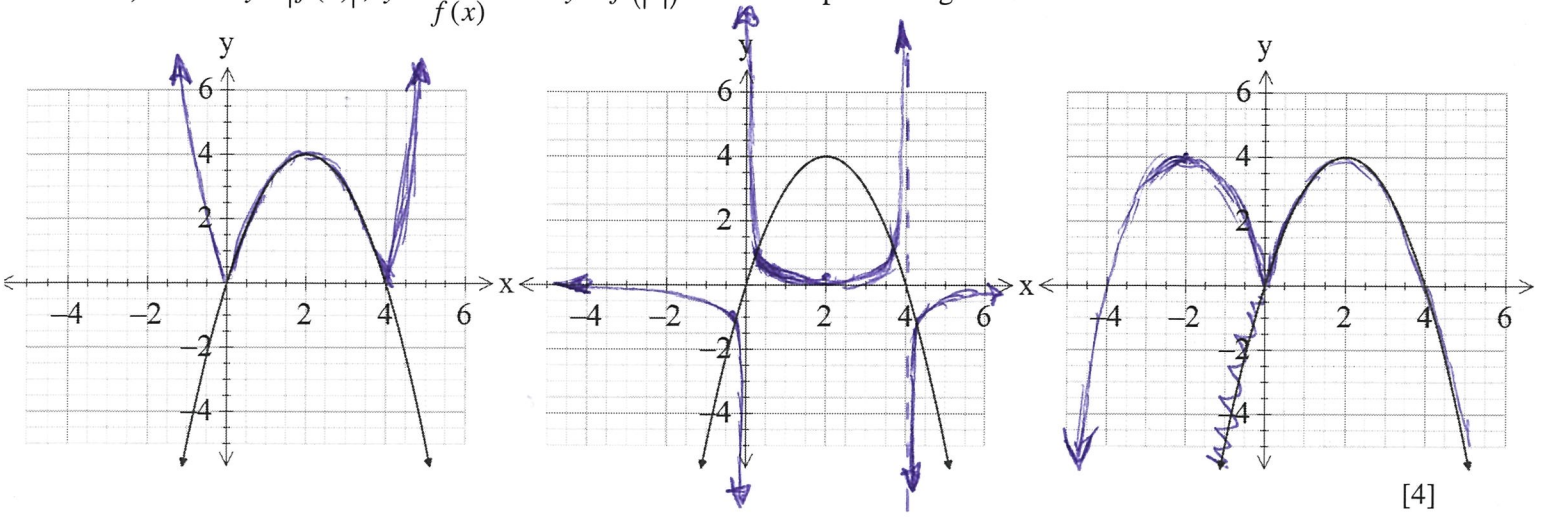
[4]

10

3. [9 marks]

The function $f(x)=4x-x^2$ is represented by the graph of $y=f(x)$ shown on each set of axes provided.

a) Draw $y=|f(x)|$, $y=\dfrac{1}{f(x)}$ and $y=f(|x|)$ on these separate diagrams:



[4]

The domain of $f(x)=4x-x^2$ is restricted to $\{x : x \in \mathbb{R}, x \le k\}$ so that $y=f^{-1}(x)$ can be defined as a function.

b) Determine the largest possible value of k

[1]

2 i.e. $k \le 2$

c) Define $y=f^{-1}(x)$ and specify its domain and range.

[4] 5

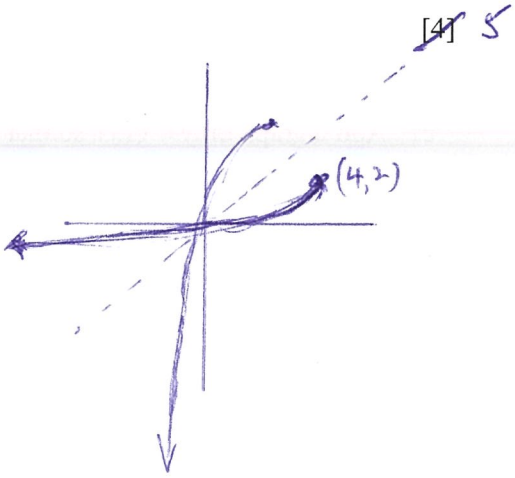
if $y = 4x - x^2 = 4 - (x-2)^2$

$\therefore x = 4 - (y-2)^2$

$\therefore (y-2)^2 = 4 - x$

$\therefore y - 2 = \pm\sqrt{4-x}$

$\therefore y = \pm\sqrt{4-x} + 2$



Inverse has $y = -\sqrt{4-x} + 2$ for $x \le 4$ (domain), $y \le 2$ (range) as req'd f^{-1}.

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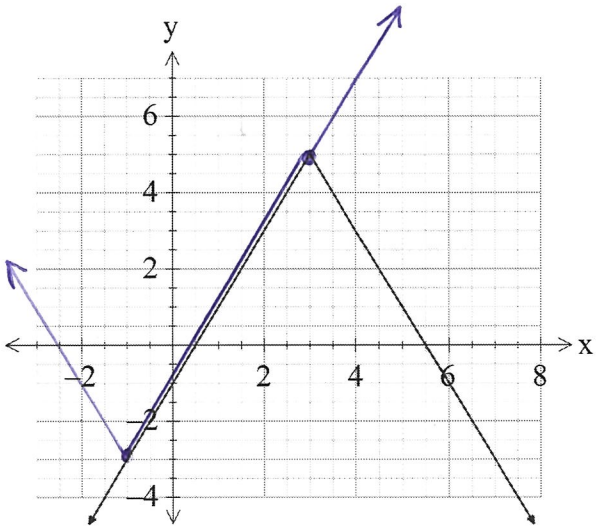
4. [7 marks]

The graph of $y = f(x)$ for $f(x) = a|x+b|+c$ has a y-intercept of $(0,-1)$ and a maximum point at $(3,5)$, as shown.



a) Evaluate a, b and c. [3]

$c = 5$

$b = -3$

$a = -2$ (gradient)

b) For which value(s) of d does $|f(x)| = d$ have exactly four solutions? [2]

$0 < d < 5$

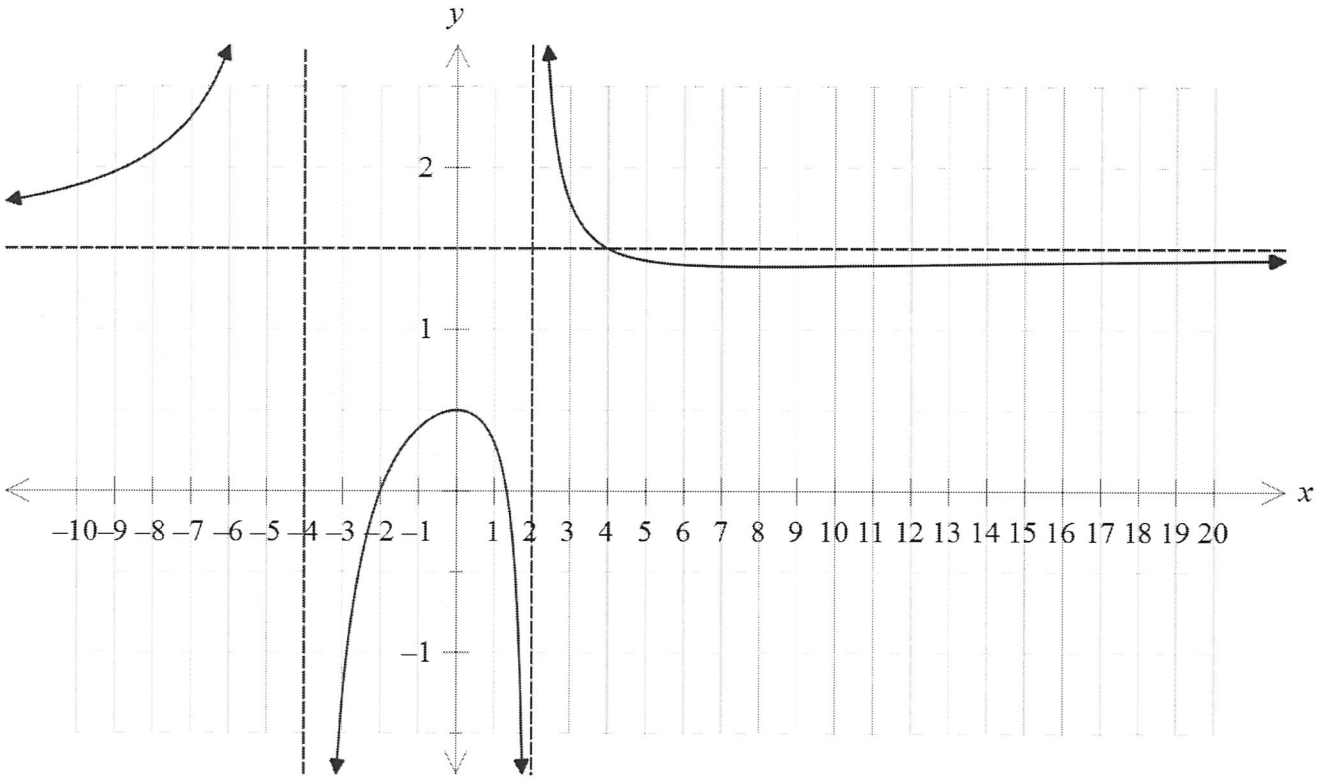
to the axes

c) Add a graph of $y = g(x)$ so that $\{x : x \in \mathbb{R} \text{ and } f(x) = g(x)\} = \{x : x \in \mathbb{R} \text{ and } -1 \le x \le 3\}$ [2]

$y = 2|x+1| - 3$ is such a function $g(x)$.

5. [7 marks]

This graph represents a function of the form $y = f(x) = \dfrac{ax^2}{(x+b)(x-c)} + d$



The asymptotes are as shown and the unmarked x intercept is $\left(\frac{4}{3}, 0\right)$.

(a) Determine the values of the constants a, b, c and d.

$b = 4$

$c = 2$

$d = 0.5$ (y value of TP)

$a = 1$

[4]

(b) What is the exact range of $y = f(x)$?

$\mathbb{R}$, $y \leq 0.5$ or $y \geq$ T.P. at $(8, 1.38\overline{8})$

i.e. $\mathbb{R}$, $y \leq 0.5$ or $y \geq \frac{25}{18}$

[3]

$x = 1.3\dot{8}$

$10x = 13.8\dot{8}$

$\therefore 9x = 12.5$

$\therefore x = \frac{25}{18}$